

IIT Mandi  
Proposal for a New Course

Course Name: Introduction to Stochastic Problems in Physics

Course Number: PH-706

Credit: 3-0-0-3

Prerequisites: Masters Level Courses in Mathematical Physics, introductory statistical mechanics course

Students intended for: Ph D

Elective or Compulsory: Elective

Semester: Odd and Even

Course Outline:

Introduction random walk in 1D, Mean values of random walk problem, Probability distribution for large  $N$ , Binomial and Gaussian distributions. Probability distribution of many variables, continuous probability distributions. General calculation of mean values for the random walk. Example of random walk problem, freely joined model for polymers, Gaussian chains.

Historical introduction to stochastic process. Einstein's formulation of the random walk. Comparison between ordinary and stochastic differential equation. Differential equation of probability – the diffusion equation from – random walk – kinetic arguments. Definition of the diffusion coefficient.

Langevin equation for a Brownian particle – average velocity and mean square displacement. Formal solution. Probabilistic approach to stochastic process. Birth and death process – master equation. Noise in the electronic system – shot noise and Johnson noise. Poisson distribution – formulation of the differential equation. Limitation of ordinary calculus.

Definition of a stochastic variable, probability distribution, probability density distribution. Transformation between stochastic variable. Characteristic function, moments and cumulants. Stochastic process of many variables. Conditional probability density, cross correlations, Multivariate Gaussian distribution.

Time depended random variables, stationary processes. Classification of stochastic process, purely random, Markov process, and non-Markovian process. Chapman Kolmogorov equation-Weiner Khinchine theorem.

Langevin equation revisited – velocity autocorrelation function- mean square displacement. Maxwell Boltzmann distribution from moments of velocity. Ornstein Uhlenbeck process, Green's function solution, correlation function, moments, solution by Fourier transformation. Non-linear Langevin equation- Kubo oscillator. Drift and diffusion coefficients – Kramer's Moyal expansion coefficients- Ito and Stratonovich's definitions of stochastic calculus.

Fokker-Planck equation- Kramers Moyal Forward and backward expansion and equivalence. Fokker-Planck equation for one variable. Application of truncated Kramer's Moyal expansion. FP equation of many variables with examples. Methods of solution of FP equation of one variable.

Discussion of Kramers problem of escape over barrier. Master equations. Probabilistic approaches molecular systems. BBGKY hierarchy. Boltzmann equation. Quantum stochastic processes. Master equation approaches to density matrix. Linear response theory – fluctuation dissipation theory.

Text & Reference Books:

- 1) The Fokker-Planck Equation Methods of solution and applications by H Risken, Springer
  - 2) Stochastic Methods: A Handbook for the Natural and social science by C Gardiner, Springer
  - 3) Synergetics : An introduction by Herman Haaken, Springer
  - 4) An introduction to statistical Communication theory by D Middleton, Peninsula Publishing
  - 5) Collected papers in noise and stochastic process by Nelson Wax, Dover
  - 6) Fundamentals of Statistical and thermal physics by F Reif, McGraw Hill
  - 7) Theory of polymer dynamics by M Doi and S F Edwards by Oxford University Press.
  - 8) An introduction to probability theory and its applications I & II by W Feller, John Wiley & Sons.
  - 9) An introduction to stochastic processes and non-equilibrium statistical physics by H S Wio, World Scientific
-